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K-SPLINE: A NEW CURVE FOR ADVANCED HULL MODELLING[1]

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Abstract. The k-spline is introduced and the underlying equations presented. These equations yield four parameters, including the area coefficient. The k-spline is inherently smooth and convex, which reduces the need for heuristic computing. The k-spline equations facilitate input parsing, which permits a user-friendly interface. A family of light displacement sailboat hulls is presented which have k-spline sections, a k-spline centreplane curve, and an overhanging transom stern. These hulls are modelled by applying a metasurface to a framework of transverse sections and longitudinal curves. These variants are created by manipulating longitudinal parameter curves which define some of the k-spline parameters. The area coefficients do not change between variants. A practical application is demonstrated, in which the underwater shape of a sailboat hull can be modified, without affecting its hydrostatic characteristics in the upright condition. All variants have practically identical displacement, LCB, and LCF, in the upright condition. In the heeled condition, the waterlines and the position of the centre of flotation differ between variants. The overhanging stern leads to a small but usually negligible impact on upright displacement, LCB, and LCF, between variants. When the hull has an immersed transom stern, manipulating the k-spline parameters does not affect the upright displacement, LCB, and LCF. It is proposed that such a family of hull shapes allows a designer to investigate potential performance differences using computational fluid dynamics and/or tank testing. This is only one of numerous possible applications for the k-spline. The k-spline has been implemented in an advanced hull modelling application.

NOMENCLATURE

α	Angle of deadrise
a_2	Floor factor
a_i	Coefficient
As	Section area
ACC	Area coefficient curve
b	Breadth at DTL
BFC	Bilge factor curve
Ca	Coefficient of section area
CL	Centreline
CPC	Centreplane curve
DEC	Deck edge curve
DRC	Deadrise curve
DTL	Datum line
FFC	Floor factor curve
h	Draft at CL
LWL	Length on the waterline
LCB	Longitudinal centre of buoyancy
LCF	Longitudinal centre of flotation
LOA	Length overall
m	Bilge factor
p_3	K-spline index
p_m	Maximum index
s	Deadrise
SAC	Curve of areas, or section area curve
t	Parametric variable
y	Athwartships co-ordinate
z	Vertical co-ordinate

1. INTRODUCTION

Parametric hull modelling offers the ability to develop a family of candidate hull shapes by varying key shape parameters. These hull shapes are then compared against mission-specific performance requirements, such as resistance, stability, and sea-keeping, under various weather and sea conditions. The variant which offers the best balance of performance criteria is selected for further development.

Implementing such a system requires software that facilitates parametric variation of the hull model: a parametric modeller.

One established approach to parametric hull design is based on a skeleton, or framework, of fair curves such as the centreplane curve CPC, the datum line DTL, and a set of section shapes. The framework defines a hull shape by specifying points and curves on the actual hull surface. This approach offers close control of key parameters, such as centre of buoyancy. It is quite different to modelling strategies that use “control points” which are indirectly related to the hull surface.

The construction of a framework-based model requires a diverse toolbox of fair curves and surfaces. Harries and Abt (1999) introduced the f-spline [1], which defines a fair curve by optimising a b-spline for fairness criteria, and the lofted surface [1], which applies fair skinning interpolation to convert the framework into a fair surface. More recently, Friendship Systems GmbH [2] have developed a more powerful type of faired surface, the metasurface. Like the lofted surface, the metasurface applies a faired b-spline surface to a framework of smooth curves. The metasurface allows a framework to be skinned in several sections, for example, one metasurface can be used to skin the main body of a hull, and a second to skin the bow. The metasurface software ensures a smooth transition from one surface to the next, by ensuring that the both surfaces have equal tangent angles along the boundary. These basic tools have made framework-based hull modelling a practical reality.

This paper introduces a new curve which complements existing parametric design tools. Designated the k-spline, it describes a smooth, convex curve suitable for the underwater sections and lower topsides of a wide variety

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1Updated 26 May 2009

of hull forms. It is also useful for longitudinal curves, especially the centreplane curve (CPC) of typical cruiser-racer and racing sailboat hulls, including multihulls.

A very useful characteristic of the k-spline is that it is possible to determine in advance, whether a set of parameters will lead to a valid curve. This is possible because the underlying equations yield a set of inequalities (equations (12) to (15)) defining a valid range for the value of each parameter, in terms of the other parameters.

This is especially useful if the modelling software is required to automatically generate a family of variants based on an initial hull model. If a particular set of parameter values would not yield a valid hull, the software can immediately recognise the problem, skip the offending variant and move onto the next. The software is not required to perform a long-winded series of trial and error computations before abandoning the invalid variant.

Another useful characteristic is that the k-spline can extend beyond the two points which define its dimensions and position. This feature is especially useful for the centreplane curve of a sailboat hull with an overhanging stern.

2. DESCRIPTION OF THE K-SPLINE

The k-spline defines a hull section with respect to the centreline CL, (where y = 0) and the datum waterline, DTL (where z = 0).

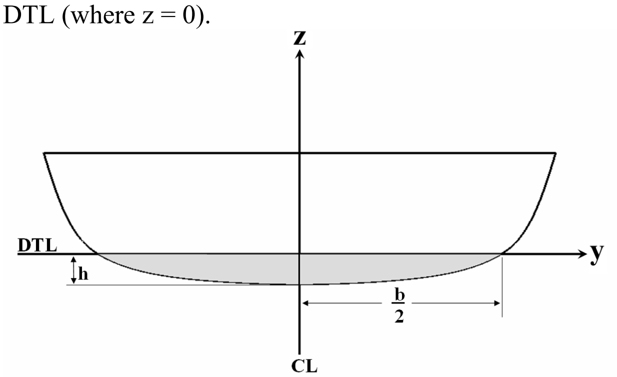

Figure 1: Typical hull section.

The k-spline equations are developed from the basic relationship between breadth, draft, area coefficient, and section area (see Figure 1):

$$As = b\ h\,Ca \qquad (1)$$

The design of the k-spline begins with the following general equation:

$$f(y) = \sum_{i=1}^{n} a_i |y|^{p_i} \qquad (2)$$

Equations of this form are readily integrable, which means it is possible to derive a simple equation describing the area coefficient. Also, note that equation (2) is a function of the absolute value of y. This means that it is symmetrical about y = 0.

The curve is normalised so that when y = 1, f(y) = 1. This is achieved by setting:

$$\sum_{i=1}^{n} a_i = 1 \qquad (3)$$

To define a k-spline of the third degree, the following substitutions are made in equation (3):

$a_1 = a_1$
$p_1 = 0$
$p_2 = mp_3$

This yields the following parametric equations:

$$y(t) = \frac{b}{2}|t| \qquad (4)$$

$$z(t) = h\left(s|t| + a_2|t|^{mp3} + (1 - s - a_2)|t|^{p3} - 1\right) \qquad (5)$$

Note that when t = 0, the gradient in Cartesian co-ordinates is equal to s, that is, the deadrise gradient = s. An expression for the area coefficient can be obtained by integrating equations (4) and (5). In Cartesian co-ordinates, this is:

$$Ca = 1 - \left(\frac{s}{2} + \frac{a_2}{mp_3 + 1} + \frac{(1 - s - a_2)}{p_3 + 1}\right) \qquad (6)$$

Equation (6) defines the index, p_3, in terms of the k-spline parameters Ca, s, a_2, and m. If the parameters are constants, equation (6) is a quadratic function of p_3. Solving for p_3 yields:

$$p_3 = \frac{-\beta_1 \pm \sqrt{\beta_1^2 - 4\beta_0\beta_2}}{2\beta_0} \qquad (7)$$

Where:

$$\beta_0 = m\left(1 - Ca - \frac{s}{2}\right) \qquad (8)$$

$$\beta_1 = 1 - Ca(m + 1) + \left(\frac{s}{2} + a_2\right)(m - 1) \qquad (9)$$

$$\beta_2 = \left(\frac{s}{2} - Ca\right) \qquad (10)$$

For any set of parameter values (Ca, s, m, a_2), equation (7) has two solutions. Practical k-spline software must determine if equation (7) has real solutions, that is, it must determine whether $\beta_1^2 > 4\beta_0\,\beta_2$.

To ensure that the curve is always convex, the indices are always greater than one. Also, from the practical perspective, it is also necessary to specify a maximum value for p_3. This avoids excessively large values for p_3, which would cause a computer system to overflow.

$$1 < p_3 \le p_m \qquad (11)$$

If equation (7) has two real solutions, the software selects the numerically greater solution, provided that it also satisfies equation (11).

The design of the k-spline ensures that each of its parameters (Ca, s, m, a_2) can range between zero and one. However, they must satisfy the following constraints:

$$0.5 < Ca \le \frac{p_m + s(1 - p_m)/2}{(p_m + 1)} \qquad (12)$$

$$0 \le s \le 1 \qquad (13)$$

$$0 \le a_2 \le (1 - s) \qquad (14)$$

$$\frac{1 - Ca - (s + a_2)/2}{Ca - (s + a_2)/2} < m \le 1 \qquad (15)$$

Equations (13) and (14) are derived from equation (6). Equations (12) and (15) are derived from equations (7) and (11).

Although the k-spline is undefined for Ca=0.5, it is possible to programme a computer system to draw a section with this value. It would be a straight line, that is, it would represent a section whose underwater shape is triangular. It is acceptable for the software to make this assumption because the closer Ca gets to 0.5, the closer the resulting curve resembles a straight line.

In practical k-spline software, equations (12) to (15) are used to check if a set of parameter values will produce a valid curve, and to reject parameter values which do not. For example, as Ca approaches 1.0, the k-spline approaches a rectangle. A rectangular section would have s=0 and Ca=1. Substituting these values into equation (8) yields $\beta_0 = 0$, resulting in division by zero in equation (7). This means that it is not possible to define a k-spline with an area coefficient of one.

Because the k-spline is defined directly from the parameters, the software yields a smooth curve without the need for heuristic (or "trial and error") processing. To determine the validity of a k-spline, the software evaluates equations (7) to (15). If the input parameter values satisfy equations (7) to (15), the system draws a smooth curve. If the input parameter values do not satisfy equations (7) to (15), the system rejects the inputs. Computer software can exploit this feature in at least two ways.

If a designer is manually developing a hull shape, and they specify a parameter value which is not valid, the software can ignore the change and warn the designer that this is not a valid request.

If the software is automatically creating a family of hulls, and it encounters a variant with an invalid combination of parameter values, the software can skip that variant and go to the next one.

In contrast, an iterative process is necessary to fair other types of spline curves. The system evaluates a set of fairness equations, adjusts the curve, re-evaluates the fairness equations, and repeats the process until the curve is as fair as the computer can make it. Only after it has finished this iterative fairing process can a computer determine whether a set of inputs will produce a valid curve.

The k-spline reduces the demand on computing resources by eliminating this iterative fairing process. It is important to note, however, that the process of adding a faired surface to the basic framework (the "skinning process") tends to demand more computing resources, than does the modelling of the framework's basic curves. Notice that the k-spline equations are valid for t>1. This means that k-spline curves can extend beyond the points from which they are defined. This is not generally true for b-spline curves [3].

Notice also that the k-spline indices are not necessarily integers. Most practical numerical techniques, such as b-spline curves, are based on polynomials with integer values: 1, 2, 3…

The k-spline is based on a polynomial in which the indices are real numbers. That is, p_3 can take any value between 1 and p_m. It is not necessarily an integer. Because the k-spline is not based on traditional polynomials, indices, it can be seen as the result of a separate line of development from that which produced tools such as the b-spline [3]. B-splines are constructed from traditional polynomial curves.

However, fair skinning systems such as the lofted surface [1] and the metasurface make it practicable to use k-spline sections in fully integrated CAD/CAM systems.

3. EFFECT OF PARAMETERS

Figures 2–4 illustrate the effect of varying the bilge factor, floor factor, and deadrise on the shape of a k-spline section with Ca = 0.75.

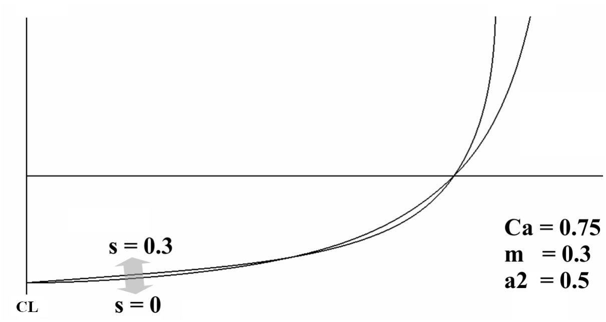

Figure 2: Effect of varying deadrise on k-spline section.

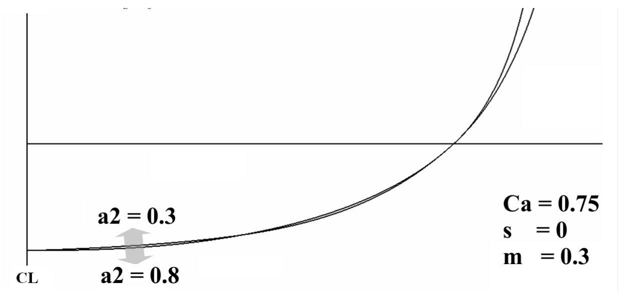

Figure 3: Effect of varying floor factor on k-spline section.

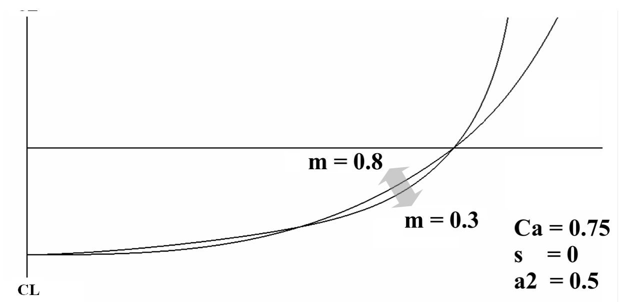

Figure 4: Effect of varying bilge factor on k-spline section.

Note that the deadrise parameter (s) is defined in algebraic terms. It is the slope of the normalised curve when t=0.
In some projects it may be preferable to specify deadrise as an angle. The relationship between the deadrise parameter (s), and the deadrise angle (α) is simply derived from equations (4) and (5) by taking the value of the derivative when t = 0:

$$\alpha = \tan^{-1}\left(s \frac{2h}{b} \right) \qquad (16)$$

4. APPLICATION TO A FRAMEWORK

Figure 5 illustrates a framework built around a k-spline centreplane curve (CPC). The underwater parts of the sections are also k-splines. The datum line (DTL) and deck edge curve (DEC) are f-splines [1].

The k-spline parameter values are defined by longitudinal f-spline curves (ACC, BFC, DRC, and FFC). Each of these four parameter curves defines the value of the relevant parameter at any position along the length of the hull.

Notice that the values of all section parameters are defined at every longitudinal position along the hull. This means that it is possible to draw a section curve at any longitudinal position.

This model was skinned with a lofted surface [1].

To illustrate the underlying framework, the starboard skin has been omitted from Figure 5. DRC and FFC have also been omitted for clarity. In Figure 5, the deadrise (s) is zero along the entire length of the hull, and the floor factor (a_2) is constant along the entire length.

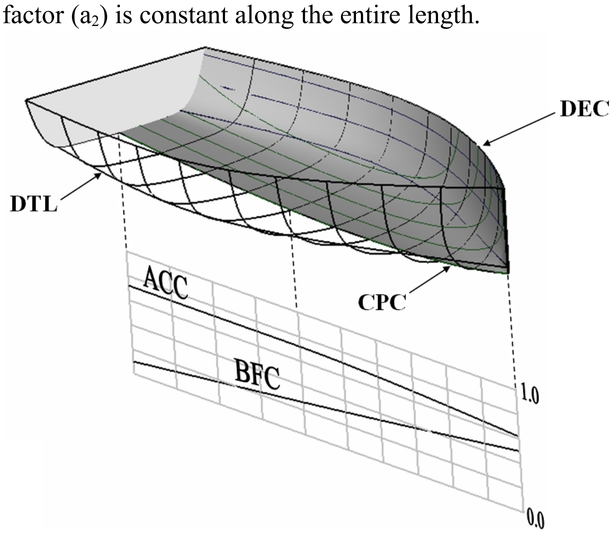

Figure 5: A simple framework based on k-spline curves.

Figure 6 illustrates the basic section curve used in the model of Figure 5. Notice that the k-spline extends slightly above the DTL. An f-spline [1] defines the upper part of the curve. The tangent angle and curvature of the f-spline segment are equal to those of the k-spline at the transition. This ensures a smooth transition.

The height of the transition point can be anywhere above DTL, at the designer's discretion. Its vertical position is defined by a longitudinal parameter curve similar to those which define the k-spline parameters.

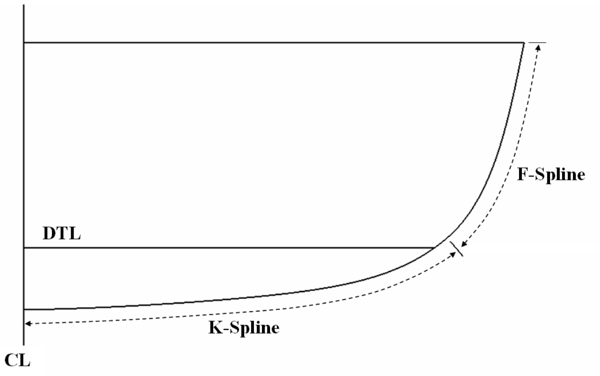

Figure 6: Section curve used in Figure 5.

5. A PRACTICAL APPLICATION: UPRIGHT AND HEELED WATERLINES AND TRIM

The k-spline and parametric hull modelling techniques can address a wide variety of design challenges.
One such challenge is the conflicting requirements of upwind and downwind sailing. The flat and level (or upright) condition resembles downwind sailing conditions, but for many sailboats it is almost entirely irrelevant upwind. What matters upwind, is how the hull performs when it is heeled.

This problem would be simplified if there were a practical method of adjusting the heeled hydrostatic characteristics of a hull, without changing its upright hydrostatics. It is particularly difficult to develop such a technique that works for hulls with flat aft sections and overhanging ends.

Equation (1) suggests a way to tackle this challenge. A family of hulls, all of which have the same design waterline, centreplane curve (CPC), and area coefficient curve (ACC), will have identical curves of areas (SAC) and displacement volumes. They will also have identical values of LCB and LCF in the upright condition.

This fact can be exploited with k-spline sections. The parameters s, a_2, and m, are independent of Ca within the limits imposed by equations (12)–(15). This means that the shape of a k-spline section can be changed without affecting its area coefficient.

The area coefficient is specified with respect to a horizontal datum line DTL. If this datum line coincides with LWL, then the underwater shape can be changed without affecting the displacement, LCB, or LCF, in the upright condition.

To demonstrate how this might be applied in a practical situation, a family of six hulls was created by varying the values of the k-spline parameters of a base hull.
The framework used for this experiment is a more sophisticated version of the framework shown in Figure 5. The sections are k-splines below DTL and f-splines above, (Figure 6) with the k-spline/ f-spline transition at the datum line. The values of the k-spline parameter curves ACC, BFC, DRC, and FFC are specified at three longitudinal reference stations, aft, mid, and fwd (see Figure 5 and Table 2). Intermediate values are specified by f-spline curves. CPC consists of k-spline underwater curves blended into the straight stem with a "fillet" curve, which is a variant of the f-spline. Deadrise is zero. The model was skinned with two metasurfaces.
All members of the family have the same basic dimensions as the base hull, "hull zero" (Table 1).

Table 1: Dimensions of all hulls.

LOA	17.167 m
LWL	15.678 m
BWL	4.67 m
Draft	0.53 m
Displ	14.973 m^3
Cp	0.529
LCB	46.77 % LWL[2]
LCF	42.12 % LWL
Ca aft	0.77
Ca mid	0.77
Ca fwd	0.68
m fwd	0.96
a_2 fwd	0.6

Hulls 1 to 3 were created by varying the values of m and a2 at the aft and mid stations, while holding all parameter values constant at the bow (Table 2). In other words, all of these hulls have similar bows, but their middle and aft bodies are different.

The hull modelling and hydrostatic calculations were done with the Friendship Framework v1.1, running on Windows Vista. The computer was a 32-bit 1.86 GHz Toshiba Satellite M01 notebook. It took about twelve seconds to generate hulls 1 to 3.

Table 2: Parameter values across the family.

Hull #	m (aft & mid)	a2 (aft & mid)	Overall breadth (m)
0	0.2	0.3	5.304
1	0.2	0.7	5.033
2	0.8	0.3	5.973
3	0.8	0.7	5.967
4	0.8	0.3	5.304
5	0.8	0.7	5.304

The overall breadth of hulls 0 to 3 was not constrained. This meant that when the k-spine parameters were changed, the overall breadth across the deck also changed.

Hulls 4 and 5 have had their overall breadth reduced so that they are the same width as hull 0. This breadth reduction affects the shape of the f-spline part of their sections, but not the k-spline part. Because the k-spline – f-spline transition is above DTL, the underwater shapes

[2] Measured from the intersection of LWL and CPC.

of hulls 4 and 5 in the upright condition are identical to those of hulls 2 and 3 respectively.

To assess the effect of parametric changes on the heeled underwater shapes of these hulls, their hydrostatic characteristics were calculated at a heel angle of thirty degrees. Each hull was trimmed and its immersion depth adjusted so that the heeled LCB was equal to the upright LCB. The heeled displacement volume was equal to the upright displacement. No allowance was made for the influence of sail forces or changes to crew positions.

Table 3: Hull characteristics at 30 degrees heel

Hull #	Trim angle (degrees)	LCF (% LWL)	LCF (metres)
0	-2.20	46.90	7.354
1	-2.03	47.13	7.389
2	-2.61	46.32	7.262
3	-2.61	46.33	7.264
4	-2.35	46.47	7.285
5	-2.35	46.47	7.286

This comparison illustrates the relative influence on the section shape, of the bilge and floor factors. For example, hulls 0 and 4 have the same floor factor and overall breadth, but different bilge factors. At thirty degrees heel, hull 4's LCF is 69 mm further aft than that of hull 0 (Table 3). Figure 7 illustrates the difference between the heeled waterlines of these two hulls.

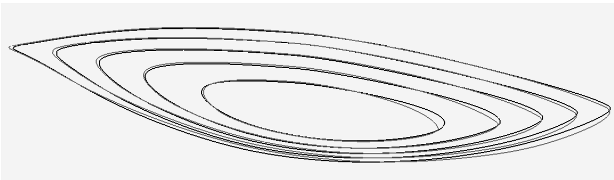

Figure 7: Heeled waterlines of hulls 0 (black) and 4 (grey).

The floor factor has a more subtle effect. Hulls 0 and 1 have the same bilge factor but different floor factors. There is a noticeable difference between the angle of the topsides at the waterline, and the underwater section shape. This affects the shape of the topsides f-spline, and is particularly apparent in the outer buttocks. (Figures 8 and 9).

However, the floor factor has only a small influence on the shape of the heeled waterlines. Hulls 4 and 5, for example, have different floor factors but the same overall breadth. Their heeled LCFs and waterlines are very similar (Table 3).

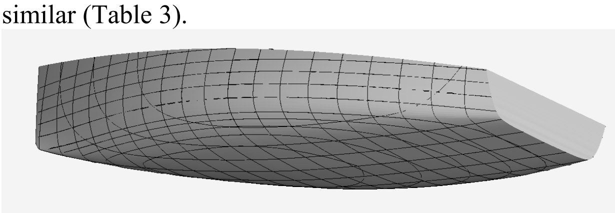

Figure 8: Hull zero.

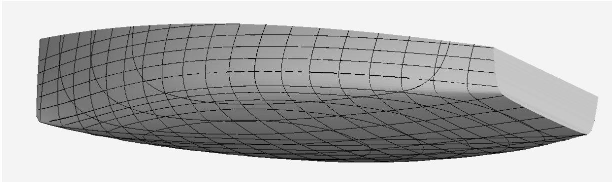

Figure 9: Hull one.

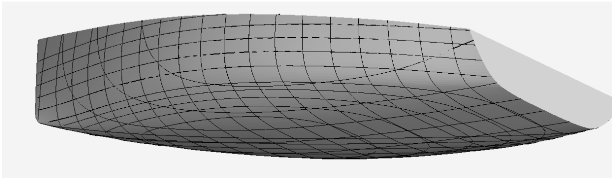

Figure 10: Hull three.

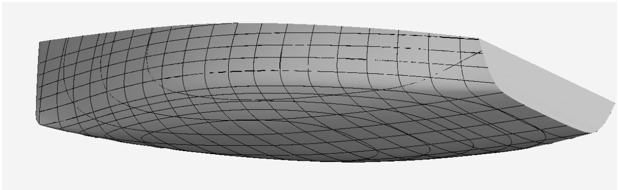

Figure 11: Hull five.

The trim angle is also affected by changes to the k-spline parameters, especially the bilge factor. Figures 12 to 14 illustrate the fore and aft overhang of several hulls at thirty degrees heel.

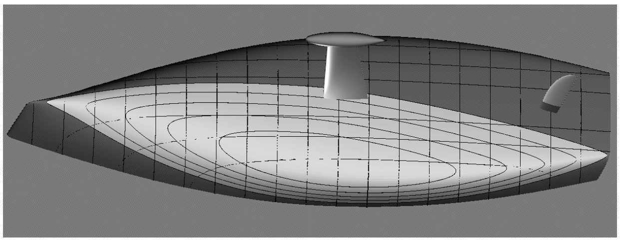

Figure 12: Fish's eye view of hull one at 30 degrees heel.

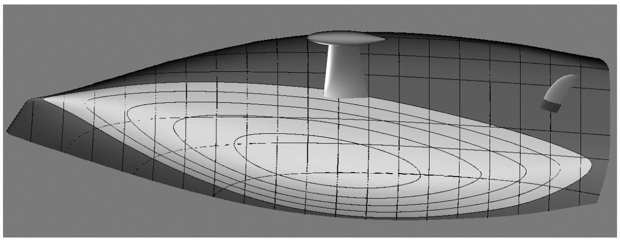

Figure 13: Fish's eye view of hull two at 30 degrees heel.

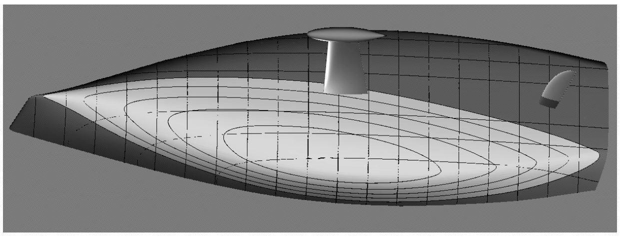

Figure 14: Fish's eye view of hull four at 30 degrees heel.

6. EFFECT OF OVERHANGS ON ACCURACY

To accommodate overhanging ends, the framework has a datum line which curves upward slightly at the ends (Figure 15).

The curved datum line overcomes three complications introduced by overhanging ends: first, the angle between the aft end of the load waterline depends on the deadrise angle and the angles of the LWL and CPC; second, the curvature of the aft end of the waterline is affected by the curvature of the sections, the LWL, and the CPC, at the aft perpendicular; and finally, since the overhang is above the load waterline, LWL is unsuitable for the horizontal reference axis of the k-spline sections. These complications make it very difficult to design a section shape that precisely fits the ends of the hull, if the datum line coincides with the LWL.

Because the datum line is not positioned exactly on LWL, the ends of the design waterplane change their shape when the k-spline parameters are changed. This affects the upright hydrostatics, however, the effect is very small.

Table 4 lists the upright hydrostatics for hulls 0 to 5, and the difference between largest and smallest values, as a percentage of the mean value across the family of hulls. Table 4 also shows the moment of the LCB about the aft end of LWL (x = 0). The difference between the highest and lowest moment is 0.02353 tonne-m, which is approximately equivalent to moving a 1.3 kg weight from one end of the boat to the other.

Table 4: Upright hydrostatics of hulls 0 to 5

Hull	Volume (m^3)	LCB (metres)	LCF (metres)	Moment (tonne–m)*
0	14.9728	7.33333	6.59670	109.80048
1	14.9730	7.33332	6.59940	109.80180
2	14.9804	7.33118	6.60652	109.82401
3	14.9803	7.33116	6.60648	109.82298
4	14.9804	7.33118	6.60650	109.82401
5	14.9805	7.33120	6.60648	109.82504
Spread**	0.051%	0.030%	0.149%	0.02353

* Fresh water.
** Percentage of family average.

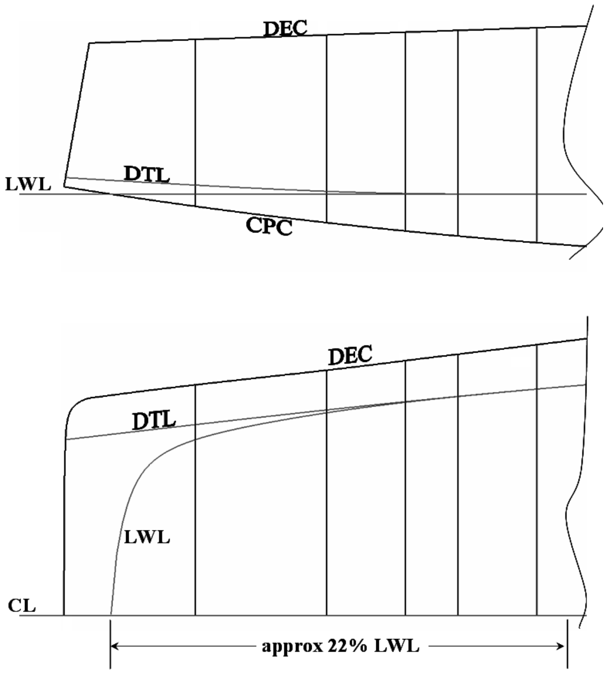

Figure 15: Aft part of the framework, showing the curved datum line (DTL).

7. RADICAL TRANSFORMATIONS

The k-spline can also be used to radically transform a hull shape.

To illustrate the potential, a new hull was developed from hull zero by changing the k-spline parameters, raising the k-spline/f-spline transition point, extending the aft overhang, narrowing the stern and altering some details of the bow and the datum line.

Hull 6 has the same LWL and BWL as hull 0, however, its maximum draft has been increased to 0.6m. Displacement volume is 14.972 m^3, and prismatic coefficient is 0.5. The k-spline parameters (Table 5) are constant along the length of the hull.

Notice that hull 6 is radically different from hulls 0 to 5, even though it was created by changing only a few hull parameters.

Table 5: Dimensions of hull six.

LOA	20.532 m
LWL	15.678 m
BWL	4.67 m
Draft	0.60 m
Displ	14.972 m^3
Cp	0.50
Ca	0.68
s	0.20
m	0.12
a_2	0.80

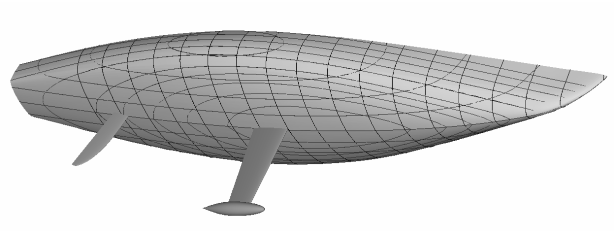

Figure 16: Hull six.

8. K-SPLINES OF HIGHER DEGREE

The author is investigating k-splines with $i>3$. A fourth degree k-spline in which $i=4$ and $p_2=2$ is particularly interesting because it offers direct control of the curvature at the centreline.

Higher degree k-splines are not available in commercial hull design software.

9. IMPLEMENTATION

The k-spline has been implemented in a commercial hull modelling application, the Friendship Framework®. This software incorporates the hull modelling features used in this paper. The performance of a hull, or a family of hulls, can be evaluated by using the optional computational fluid dynamics software which is available as an add-on to the Friendship Framework®.

The Friendship Framework® has industry-standard interfaces which allow a designer to transfer the hull model into other programmes. For example, the model can be exported to industry-standard CAM systems, for example, to facilitate the construction of models for tank testing.

Finished hull models can be exported to detail drafting software for final design, using standard output formats such as IGES.

10. CONCLUSIONS

The k-spline complements existing parametric hull design tools. It is particularly useful for the underwater sections of all round-bilge hull forms. Used as part of a hull's centreplane curve, it simplifies the design of overhanging sterns.

This paper has demonstrated how the k-spline can be used to change the underwater shape of a sailboat hull in a structured way. Hulls 0 to 5 have different underwater shapes, but they all have the same upright hydrostatic characteristics.

Such a family could be subjected to a CFD analysis or tank testing designed to identify the hull which best fits a particular design brief.

The k-spline also facilitates radical hull transformations. Hull 6 was created by varying a few parameters of hull 0, and yet hull 6 looks noticeably different from hulls 0 to 5.

The purpose of this work has been to introduce the k-spline and illustrate a potential application.
The k-spline has the potential to address other practical design challenges. The author does not claim to have thought of all possible applications.

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